

SUPPLEMENTAL MATERIALS

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Social Equity of Bridge Management

Cari Anne Gandy, Daniel Erian Armanios, and
Constantine Samaras

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This Supplemental Materials document details the ordered probit model and demonstrates the robustness of our results with alternate model specifications. We include models with alternate census years for the race and ethnicity indicators, alternate thresholds for the race and ethnicity indicators, and the Social Vulnerability Index (Centers for Disease Control 2020) metric as an alternative to the disadvantaged community indicator (Council on Environmental Quality 2020).

CODE, DATA, AND ADDITIONAL INFORMATION

The final panel dataset, code used to prepare the panel from raw data, code to run the models (including the robustness checks) and produce Latex tables, and more detailed data descriptions, are available from the Carnegie Mellon University KiltHub Repository (Gandy et al. 2022).

ORDERED PROBIT MODEL WITH RANDOM EFFECTS

By using an ordered model, we implicitly assume that there are latent variables that describe bridge component conditions on a continuum and the discrete ratings recorded by bridge inspectors are indirect measures of this unobserved variable, V . (Washington et al. 2011; Saeed et al. 2017). We incorporate bridge-specific individual effects (η) into the ordered probit model to account for cross-sectional heterogeneity in the panel. (Washington et al. 2011; Lu et al. 2019). Therefore, for each unique bridge structure (n) in each inspection year (t), the latent variable (V_{nt}) for each component is estimated as the linear combination of the regressors (\mathbf{x}_{nt}) multiplied by the estimated regression coefficients (β_{nt}), a bridge-specific individual effect (η_n), and an idiosyncratic error term (ϵ_{nt}) (Saeed et al. 2017).

$$V_{nt} = \beta^T \mathbf{x}_{nt} + \eta_n + \epsilon_{nt} \quad (1)$$

Sources: (Saeed et al. 2017; Croissant and Millo 2018)

We used a probit model under the assumption that the cumulative distribution of the idiosyncratic term is normally distributed (Croissant and Millo 2018; Washington et al. 2011).

$$F(\epsilon) = \int_{-\infty}^{\epsilon} \frac{1}{\sqrt{2\pi}} e^{-0.5t^2} dt \quad (2)$$

Source: (Croissant and Millo 2018)

To replicate previously refined methods and reduce computational requirements, we changed our dependent variable from the 0-9 condition ratings in the NBI to four condition states: state one - failed to fair (0-5), state two - satisfactory (6), state three - good or very good (7-8), and state four - excellent (9), identical to prior work (Saeed et al. 2017). For each unique bridge structure (n) in each inspection year (t), the unobserved latent variable (V_{nt}) is mapped into these four discrete condition states (y_{nt}) in relation to a vector of threshold parameters $\mu = (\mu_0, \mu_1, \mu_2)$.

$$\begin{aligned} P(y_{nt} = 1) &= P(V_{nt} \leq \mu_0) = F(\mu_0 - V_{nt}) \\ P(y_{nt} = 2) &= P(\mu_0 < V_{nt} \leq \mu_1) = F(\mu_1 - V_{nt}) - F(\mu_0 - V_{nt}) \\ P(y_{nt} = 3) &= P(\mu_1 < V_{nt} \leq \mu_2) = F(\mu_2 - V_{nt}) - F(\mu_1 - V_{nt}) \\ P(y_{nt} = 4) &= P(\mu_2 < V_{nt}) = 1 - F(\mu_2 - V_{nt}) \end{aligned} \quad (3)$$

Sources: (Saeed et al. 2017; Croissant and Millo 2018)

Incorporating the definition of the continuous latent variable (V_{nt}) in Equation 1 into Equation 3, the conditional probability that bridge n with individual effect η_n is in condition state y_n for all (T) inspection years and all ($J = 4$) condition states is represented by Equation 4.

$$P(y_n|\eta_n) = \prod_{t=1}^T \sum_{j=1}^J (y_{nt} = j) [F(\mu_{j-1} - \beta^T \mathbf{x}_{nt} - \eta_n) - F(\mu_{j-2} - \beta^T \mathbf{x}_{nt} - \eta_n)] \quad (4)$$

Source: (Croissant and Millo 2018)

We make the assumption that the bridge-specific individual effects (η_n) are normally distributed with mean zero based on the large number of bridges sampled (Saeed et al. 2017). Assuming random effects is necessary for including time-independent explanatory variables in the model (all explanatory variables other than age and condition) (Washington et al. 2011). Assuming that η is normally distributed with standard deviation σ_η , the unconditional probability used in the ordered probit model becomes:

$$P(y_n) = \int_{-\infty}^{+\infty} \prod_{t=1}^T \sum_{j=1}^J (y_{nt} = j) [F(\mu_{j-1} - \beta^T \mathbf{x}_{nt} - \eta) - F(\mu_{j-2} - \beta^T \mathbf{x}_{nt} - \eta)] \frac{1}{\sigma_\eta \sqrt{2\pi}} e^{-0.5(\frac{\eta}{\sigma_\eta})^2} d\eta \quad (5)$$

Source: (Croissant and Millo 2018)

This integral cannot be evaluated analytically, so we use a change of variable $\omega = \frac{\eta}{\sigma_\eta \sqrt{2}}$, to get the equation in an appropriate form for approximation with Gauss-Hermite quadrature (Blevins 2022):

$$P(y_n) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \prod_{t=1}^T \sum_{j=1}^J (y_{nt} = j) [F(\mu_{j-1} - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega \sqrt{2}) - F(\mu_{j-2} - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega \sqrt{2})] e^{-\omega^2} d\omega \quad (6)$$

Source: (Croissant and Millo 2018)

The Gauss-Hermite quadrature approximation, yields:

$$P(y_n) \approx \frac{1}{\sqrt{\pi}} \sum_{r=1}^R \prod_{t=1}^T \sum_{j=1}^J (y_{nt} = j) [F(\mu_{j-1} - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2}) - F(\mu_{j-2} - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2})] e^{-\omega_r^2} d\omega \quad (7)$$

Source: (Croissant and Millo 2018)

There are four condition states, $j \in (1, 2, 3, 4)$ and we set $\mu_0 = 0$ (the threshold between states one and two) without loss of generality because our regressors (\mathbf{x}_{nt}) contain an intercept (Washington et al. 2011). As defined in Equation 3, the condition cannot be lower than state one ($\mu = -\infty$) or higher than state four ($\mu = +\infty$). Thus, only the interior thresholds between states two and three (μ_1) and between states three and four (μ_2) are estimated by the model in conjunction with the regression coefficients ($\boldsymbol{\beta}$) (Saeed et al. 2017). We implemented this method using the Panel Generalized Linear Models "pglm" package for R (Croissant 2021).

$$\begin{aligned} P(y_n) \approx & \frac{1}{\sqrt{\pi}} \sum_{r=1}^R \prod_{t=1}^T [(y_{nt} = 1) F(-\boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2}) \\ & + (y_{nt} = 2) [F(\mu_1 - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2}) - F(-\boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2})] \\ & + (y_{nt} = 3) [F(\mu_2 - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2}) - F(\mu_1 - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2})] \\ & + (y_{nt} = 4) [1 - F(\mu_2 - \boldsymbol{\beta}^T \mathbf{x}_{nt} - \sigma_\eta \omega_r \sqrt{2})]] e^{-\omega_r^2} d\omega \end{aligned} \quad (8)$$

Sources: (Croissant and Millo 2018; Saeed et al. 2017)

ROBUSTNESS CHECKS

Replication of Saeed et al. (2017, Table 5)

Our methodology follows from an ordered probit with random effects model used to analyze Indiana bridges with records from 1992 to 2014 (Saeed et al. 2017). To check our implementation of their methodology, we replicated all aspects of Saeed et al. (2017) with the exception of the high-freeze thaw cycle variable and intervention variables for past repairs, rehabilitations, and replacements based on the history of component conditions improving. Table S1 displays the results of replicating Saeed et al.'s (2017) models by subsetting our final panel to Indiana State Highway bridges (NBI variable 21: Maintenance Authority) inspected before 2015, changing the temperature threshold to 11 degrees Celsius (52 degrees Fahrenheit), and omitting the intervention and freeze-thaw cycle variables. In contrast with previous results from Saeed et al. (2017, Table 5), we did not have consistent and robust results for urban bridges across all models and we obtained positive coefficients for steel bridges and bridges over waterways. From our replication, we conclude that our selection of physical and environmental variables and our model specification is robust and consistent with previous work.

Table S1. Replication of Saeed et al. (2017, Table 5), Panel Subset of 1990-2014 Inspections of State Highway Bridges in Indiana, Comparison of Ordered Probit with Random Effects Models, Component Condition States 1-4, Warm Region Indicator Changed to 52 degrees Fahrenheit (50% of Subset). Excludes Freeze-thaw Cycles > 60 (100% of Subset), Excludes Maintenance Intervention Indicators.

	Deck	Superstructure	Substructure
Intercept	2.112 (0.025)***	2.461 (0.025)***	2.473 (0.028)***
Age, years	-0.046 (0.001)***	-0.045 (0.001)***	-0.041 (0.001)***
Interstate indicator	-0.112 (0.020)***	-0.039 (0.020)	
Urban indicator	-0.059 (0.017)***	0.084 (0.019)***	0.038 (0.022)
Average temperature > 11 deg C	0.376 (0.016)***	0.193 (0.016)***	0.186 (0.018)***
Deck protection indicator	0.459 (0.017)***		
Steel structure indicator		0.152 (0.047)**	
Bridge over waterway indicator		0.170 (0.019)***	0.093 (0.021)***
μ_1	1.298 (0.009)***	1.172 (0.009)***	1.336 (0.011)***
μ_2	4.447 (0.020)***	4.679 (0.022)***	5.064 (0.024)***
σ_η	1.035 (0.010)***	1.078 (0.011)***	1.284 (0.010)***
Log Likelihood	-45790	-40365	-35251
AIC	91598	80749	70518

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Models with Alternate Threshold Socioeconomic Variables

We chose a 50% threshold to determine whether a tract had a minority of White individuals, a majority of Black or African American individuals, or a majority of Hispanic or Latino individuals. Alternatively, we considered lowering the threshold to the national average so that more tracts meet the criteria or increasing the threshold to sixty percent which stabilizes the measure across the 2000, 2010, and 2020 Decennial census years. We replicated the models shown in Table 4 of the article using the national average threshold and find that the negative effect size is larger for the Black or African American indicator and the positive effect size is smaller for the Hispanic or Latino indicator (Table S2). Using a sixty percent threshold (Table S3), we find a somewhat smaller negative effect size for the Black or African American indicator and a similar positive effect size for the Hispanic or Latino indicator when compared to the 50% threshold variables in Table 4. In summary, for the full panel, the results for the race and ethnicity indicators are robust and consistent in direction even when the threshold for selecting these binary indicators is changed from the national average to sixty percent of the tract population.

Table S2. National Average Threshold for Race and Ethnicity Indicators, Comparison of Ordered Probit with Random Effects Models, Component Condition States 1-4, 2020 Income, Race, and Ethnicity Indicators

	Deck	Superstructure	Substructure
Intercept	1.641 (0.043)***	1.744 (0.044)***	1.469 (0.047)***
Age, years	-0.038 ($5 \cdot 10^{-5}$)***	-0.041 ($5 \cdot 10^{-5}$)***	-0.037 ($5 \cdot 10^{-5}$)***
Urban indicator	-0.118 (0.003)***	-0.039 (0.003)***	0.033 (0.004)***
Interstate indicator	-0.116 (0.005)***	-0.107 (0.005)***	-0.091 (0.006)***
Average Daily Traffic (ADT)	$-2 \cdot 10^{-6}$ ($7 \cdot 10^{-8}$)***	$-4 \cdot 10^{-6}$ ($8 \cdot 10^{-8}$)***	$-3 \cdot 10^{-6}$ ($1 \cdot 10^{-7}$)***
% ADT trucks	-0.009 ($2 \cdot 10^{-4}$)***	-0.003 ($2 \cdot 10^{-4}$)***	$3 \cdot 10^{-4}$ ($2 \cdot 10^{-4}$)
Detour length, kilometers	-0.002 ($2 \cdot 10^{-5}$)***	-0.001 ($2 \cdot 10^{-5}$)***	$4 \cdot 10^{-4}$ ($2 \cdot 10^{-5}$)
Deck protection indicator	0.218 (0.002)***		
Steel structure indicator		0.016 (0.004)***	0.099 (0.005)***
Bridge over waterway indicator		-0.071 (0.004)***	-0.131 (0.004)***
Average temperature > 18 deg C	0.139 (0.004)***	0.106 (0.004)***	0.052 (0.005)***
Annual freeze-thaw cycles > 60	-0.080 (0.004)***	-0.137 (0.004)***	-0.113 (0.004)***
Annual precipitation > 127 cm	0.020 (0.003)***	-0.064 (0.003)***	-0.074 (0.003)***
Logarithm of median income	0.103 (0.004)***	0.122 (0.004)***	0.124 (0.005)***
Black or African American > 13%	-0.108 (0.003)***	-0.120 (0.003)***	-0.175 (0.004)***
Hispanic or Latino > 19%	0.035 (0.004)***	0.076 (0.004)***	0.099 (0.004)***
μ_1	0.899 (0.001)***	0.863 (0.001)***	0.905 (0.001)***
μ_2	3.300 (0.001)***	3.306 (0.001)***	3.282 (0.001)***
σ_{η}	1.255 (0.001)***	1.293 (0.001)***	1.313 (0.001)***
Log Likelihood	-4991078	-4707984	-4766942
AIC	9982191	9416003	9533921

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table S3. Sixty Percent Threshold for Race and Ethnicity Indicators, Comparison of Ordered Probit with Random Effects Models, Component Condition States 1-4, 2020 Income, Race, and Ethnicity Indicators

	Deck	Superstructure	Substructure
Intercept	1.286 (0.042)***	1.330 (0.044)***	0.942 (0.044)***
Age, years	-0.038 ($5 \cdot 10^{-5}$)***	-0.041 ($5 \cdot 10^{-5}$)***	-0.039 ($4 \cdot 10^{-5}$)***
Urban indicator	-0.133 (0.003)***	-0.052 (0.003)***	0.017 (0.003)***
Interstate indicator	-0.122 (0.005)***	-0.111 (0.005)***	-0.097 (0.005)***
Average Daily Traffic (ADT)	$-1 \cdot 10^{-6}$ ($7 \cdot 10^{-8}$)***	$-4 \cdot 10^{-6}$ ($9 \cdot 10^{-8}$)***	$-3 \cdot 10^{-6}$ ($9 \cdot 10^{-8}$)***
% ADT trucks	-0.012 ($2 \cdot 10^{-4}$)***	-0.003 ($2 \cdot 10^{-4}$)***	-0.026 ($2 \cdot 10^{-4}$)***
Detour length, kilometers	-0.002 ($2 \cdot 10^{-5}$)***	-0.001 ($2 \cdot 10^{-5}$)***	$-1 \cdot 10^{-6}$ ($2 \cdot 10^{-5}$)
Deck protection indicator	0.215 (0.002)***		
Steel structure indicator		0.018 (0.004)***	0.101 (0.004)***
Bridge over waterway indicator		-0.065 (0.004)***	-0.121 (0.004)***
Average temperature > 18 deg C	0.138 (0.004)***	0.105 (0.004)***	0.053 (0.004)***
Annual freeze-thaw cycles > 60	-0.059 (0.004)***	-0.116 (0.004)***	-0.082 (0.004)***
Annual precipitation > 127 cm	0.003 (0.003)	-0.085 (0.003)***	-0.104 (0.003)***
Logarithm of median income	0.134 (0.004)***	0.160 (0.004)***	0.196 (0.004)***
Black or African American > 60%	-0.029 (0.008)***	-0.035 (0.008)***	-0.086 (0.008)***
Hispanic or Latino > 60%	0.087 (0.008)***	0.184 (0.008)***	0.215 (0.009)***
μ_1	0.897 (0.001)***	0.863 (0.001)***	0.965 (0.001)***
μ_2	3.297 (0.001)***	3.305 (0.001)***	3.367 (0.001)***
σ_η	1.256 (0.001)***	1.293 (0.001)***	1.296 (0.001)***
Log Likelihood	-4994105	-4709581	-4732726
AIC	9988243	9419198	9465489

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Models with Alternate Census Year Socioeconomic Variables

The socioeconomic variables are not perfectly correlated in census years 2000, 2010, and 2020. We assumed that this is mostly due to national trends and would not affect the overall distribution of these indicators. To test this assumption, we compared models with 2000 Decennial census data, 2010 Decennial census and 2006-2010 ACS data, and 2020 Decennial census and 2016-2020 ACS data. We find that the coefficients for logarithm of median household income and majority Hispanic or Latino tracts are robustly and consistently positive and the coefficients for majority Black or African American tracts are robustly and consistently negative across census years and bridge components. We show our results for superstructure condition in Table S4.

Table S4. Comparison of Census Years, Ordered Probit Random Effects Models, Superstructure Condition States 1-4

	2000	2010	2020
Intercept	1.263 (0.041)***	1.386 (0.042)***	1.330 (0.043)***
Age, years	-0.041 ($5 \cdot 10^{-5}$)***	-0.041 ($5 \cdot 10^{-5}$)***	-0.041 ($5 \cdot 10^{-5}$)***
Urban indicator	-0.054 (0.003)***	-0.052 (0.003)***	-0.053 (0.003)***
Interstate indicator	-0.110 (0.005)***	-0.110 (0.005)***	-0.111 (0.005)***
Average Daily Traffic (ADT)	$-4 \cdot 10^{-6}$ ($8 \cdot 10^{-8}$)***	$-4 \cdot 10^{-6}$ ($9 \cdot 10^{-8}$)***	$-4 \cdot 10^{-6}$ ($8 \cdot 10^{-8}$)***
% ADT trucks	-0.004 ($2 \cdot 10^{-4}$)***	-0.003 ($2 \cdot 10^{-4}$)***	-0.003 ($2 \cdot 10^{-4}$)***
Detour length, kilometers	-0.001 ($2 \cdot 10^{-5}$)***	-0.001 ($2 \cdot 10^{-5}$)***	-0.001 ($2 \cdot 10^{-5}$)***
Steel structure indicator	0.018 (0.004)***	0.018 (0.004)***	0.018 (0.004)***
Bridge over waterway indicator	-0.067 (0.004)***	-0.066 (0.004)***	-0.065 (0.004)***
Average temperature > 18 deg C	0.111 (0.004)***	0.104 (0.004)***	0.102 (0.004)***
Annual freeze-thaw cycles > 60	-0.118 (0.004)***	-0.117 (0.004)***	-0.115 (0.004)***
Annual precipitation > 127 cm	-0.085 (0.003)***	-0.083 (0.003)***	-0.081 (0.003)***
Logarithm of median income	0.164 (0.004)***	0.158 (0.004)***	0.160 (0.004)***
Black or African American > 50%	-0.054 (0.006)***	-0.059 (0.006)***	-0.046 (0.006)***
Hispanic or Latino > 50%	0.158 (0.008)***	0.163 (0.008)***	0.171 (0.007)***
μ_1	0.862 (0.001)***	0.863 (0.001)***	0.863 (0.001)***
μ_2	3.304 (0.001)***	3.306 (0.001)***	3.305 (0.001)***
σ_η	1.293 (0.001)***	1.292 (0.001)***	1.293 (0.001)***
Log Likelihood	-4710871	-4708425	-4710078
AIC	9421777	9416886	9420193

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Models with Alternate Disadvantaged Community Indicator

As an alternative to the disadvantaged community indicator currently in use by the Justice40 initiative, we ran our models with another indicator used by the federal government, the social vulnerability index (SVI) (Centers for Disease Control 2020). Specifically, we created a dummy variable for all census tracts that had at least one vulnerability flag (90th percentile) in the socioeconomic category ("Theme 1"). This indicator had a correlation of 0.44 with the disadvantaged communicator in our panel, primarily because it flagged less tracts overall. When comparing Table 5 of the manuscript with Table S5, we found that the disadvantaged community indicator (Council on Environmental Quality 2020) and vulnerable community indicator (Centers for Disease Control 2020) yielded coefficients of the same sign and a similar magnitude (≈ -0.05).

Table S5. Comparison of Ordered Probit Random Effects Models, Component Condition States 1-4, CDC/ATSR Socioeconomically Vulnerable Community, Race, and Ethnicity Indicators

	Deck	Superstructure	Substructure
Intercept	1.987 (0.004)***	2.234 (0.004)***	2.055 (0.005)***
Age, years	-0.029 ($4 \cdot 10^{-5}$)***	-0.027 ($4 \cdot 10^{-5}$)***	-0.027 ($5 \cdot 10^{-5}$)***
Urban indicator	-0.125 (0.003)***	-0.045 (0.003)***	0.024 (0.003)***
Interstate indicator	-0.115 (0.004)***	-0.110 (0.005)***	-0.096 (0.005)***
Average Daily Traffic (ADT)	$8 \cdot 10^{-7}$ ($6 \cdot 10^{-8}$)***	$2 \cdot 10^{-6}$ ($7 \cdot 10^{-8}$)***	$-2 \cdot 10^{-8}$ ($4 \cdot 10^{-8}$)
% ADT trucks	0.011 ($1 \cdot 10^{-4}$)***	-0.032 ($1 \cdot 10^{-4}$)***	-0.018 ($2 \cdot 10^{-4}$)***
Detour length, kilometers	0.001 ($2 \cdot 10^{-5}$)***	$-8 \cdot 10^{-5}$ ($2 \cdot 10^{-5}$)***	0.001 ($2 \cdot 10^{-5}$)***
Deck protection indicator	0.181 (0.002)***		
Steel structure indicator		0.018 (0.004)***	0.101 (0.004)***
Bridge over waterway indicator		-0.080 (0.003)***	-0.135 (0.003)***
Average temperature > 64 deg F	0.130 (0.003)***	0.101 (0.004)***	0.049 (0.004)***
Annual freeze-thaw cycles > 60	-0.069 (0.003)***	-0.128 (0.003)***	-0.093 (0.003)***
Annual precipitation > 50 inches	-0.002 (0.003)	-0.090 (0.003)***	-0.108 (0.003)***
Vulnerable community (CDC)	-0.050 (0.003)***	-0.057 (0.003)***	-0.049 (0.003)***
Black or African American > 50%	-0.059 (0.005)***	-0.057 (0.005)***	-0.120 (0.006)***
Hispanic or Latino > 50%	0.082 (0.005)***	0.173 (0.006)***	0.190 (0.006)***
μ_1	1.075 (0.001)***	0.843 (0.001)***	0.818 (0.001)***
μ_2	3.547 (0.002)***	3.265 (0.001)***	3.144 (0.001)***
σ_η	1.176 (0.001)***	1.297 (0.001)***	1.343 (0.001)***
Log Likelihood	-4876414	-4788638	-4886580
AIC	9752863	9577312	9773195

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

DATA AVAILABILITY STATEMENT

Some or all data, models, or code generated or used during the study are available in a repository online in accordance with funder data retention policies. Some or all data, models, or code used during the study were provided by a third party. Direct requests for these materials may be made to the provider as indicated in the Acknowledgements.

NOTATION

The following symbols are used in this paper:

- β_{nt} = Regression coefficient for bridge n in year t ;
- ϵ_{nt} = Idiosyncratic error term for bridge n in year t ;
- η_n = Bridge-specific individual effect;
- $F()$ = Cumulative normal distribution;
- j = Index for condition state;
- J = Highest condition state (4);
- μ = Interior threshold ;
- n = Index for an individual bridge;
- N = Total unique bridge structures;
- ω = Change of variable for Gauss-Hermite quadrature approximation of the models;
- $P(y_n|\eta_n)$ = Conditional probability that bridge n is in state y ;
- $P(y_n)$ = Unconditional probability that bridge n is in state y ;
- q_n = Number of records for an individual bridge;
- \bar{q} = Average number of inspections per bridge;
- Q = Total inspection records;
- r = Index for Gauss-Hermite quadrature approximations;
- R = Number of evaluations of Gauss-Hermite quadrature approximations;
- σ_η = Standard deviation of normally distributed individual random effect η ;
- t = Index for inspection year;
- T = Last inspection year for an individual bridge;
- V_{nt} = Unobserved latent variable for bridge n in year t ;
- \mathbf{x}_{nt} = Regressor for bridge n in year t ;
- y = Observed condition state

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